Transition processes in shock wave interactions

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Summary

This paper is a discussion of recent experiments in shock-wave refraction which have clarified a special type of shock outflow process appearing to have relevance to other shock interactions, and notably to shock reflection from an oblique wall. For certain incident shock strengths and angles of incidence α, the air/methane refraction problem simulates closely the situation in the troublesome range of the reflection problem, in which α lies between the value α_e at which the theoretical solutions terminate and the value α_0 that marks the onset of Mach reflection, and in which the flow deflections cannot be reconciled with theoretically permissible reflected shock strengths. In the analogous refraction cases, the reflected shock is observed to increase in strength along its length to a maximum value at the intersection point, and to be followed by a subsonic rarefaction zone which also increases in severity near the intersection. In fact, this zone appears to coalesce into a subsonic discontinuity, just at the intersection point—a feature which would contradict one of the basic assumptions of the regular reflection and refraction theories. Other refraction experiments suggest that a similar process is relevant to the Mach reflection configuration, and may account for the discrepancies in the three-shock theory for weak incident shocks.

1. Introduction

In the branch of fluid mechanics concerned with shock-wave phenomena, the problem of the reflection of a shock front from a rigid boundary has long been a matter of fundamental interest and concern. This was the first shock interaction to be studied experimentally in sufficient detail to permit a significant appraisal of the theoretical techniques available for the treatment of these non-linear problems. The earliest results from such experiments uncovered discrepancies between mathematical prediction and observed behaviour that were sufficiently acute to cast doubt on the validity of some basic theoretical assumptions. And to this day, despite considerable systematic experimental and theoretical work on this subject (Smith 1945; Fletcher, Taub & Bleakney 1951; White 1952; von Neumann 1943; Polachek & Seeger 1944; Bargmann 1945; Bleakney & Taub 1949), certain of these paradoxes have not yet been resolved, and our understanding of the reflection problem remains incomplete.

The theoretical approach to the problem is discussed in detail in most of the references cited above, and has recently been reviewed by Griffith & Bleakney (1954). For the purposes of this article it will be sufficient to recall the assumptions that underlie the formulation of the problem* (see figure 1).

(*a*) The interaction of the incident shock with the wall results in a single reflected shock travelling away from the wall into the medium behind the incident shock.

(*b*) Each of the three angular regions of flow formed by this configuration of shocks and boundary is uniform; hence the state of the gas changes only across the two shocks.

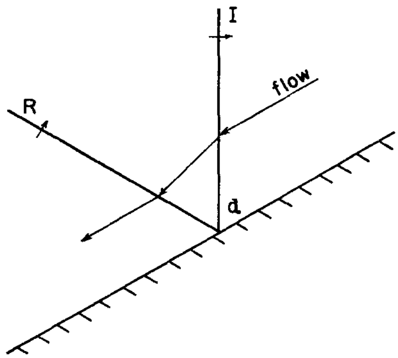

Figure 1. Regular reflection of a plane shock: incident shock I deflects flow toward wall; reflected shock R re-deflects flow parallel to wall.

(*c*) Each shock can be treated as in the standard Rankine–Hugoniot theory to relate the change in flow velocity to the pressure ratio across it.

(*d*) The net deflection of the gas flow by the two shocks is such that the flow in the region behind the reflected shock is parallel to the wall.

(*e*) The configuration is stationary when expressed in coordinates x/t, y/t.

(*f*) No energy is lost to the wall during the process.

Assumption (*b*), which will require further examination later, was introduced to simplify the analysis. Without it, one would need to include the appropriate gas-dynamical equations for the two-dimensional flow in the angular regions, to be solved simultaneously with the shock equations. This assumption has the effect, in the more general case of curved shocks

* The experiments on which this paper is based were performed in a shock tube (cf. Bleakney, Weimer & Fletcher 1949). Unless otherwise noted, all discussions and illustrations likewise refer to the shock-tube situation, in which the incident shock advances into a gas at rest. In comparing shock tube problems with the analogous wind-tunnel or free-flight situations, it should be noted that more than a simple coordinate transformation is involved. In the latter cases, boundary layers exist on all surfaces exposed to the flow; and it is with these, rather than with the surfaces themselves, that the shocks interact.

and non-uniform flow fields, of restricting the consideration to a small area surrounding the intersection point—small enough that the changes of state of the gas in the angular regions are negligible compared with the discontinuous changes across the shocks.

The assumptions listed above define a 'regular reflection' process, solutions for which can be found only for angles of incidence α smaller than some limiting angle α_e, which depends on the incident shock strength and the gas being considered (see figure 2). For larger values of α, no real solutions exist, implying that physically the assumed process cannot occur for these angles of incidence.

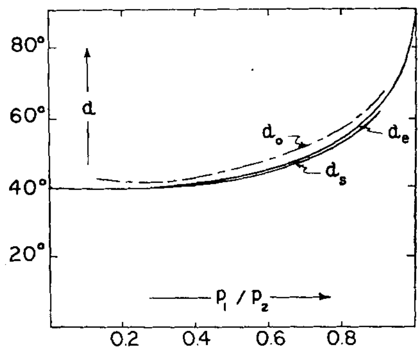

Figure 2. Critical angles of incidence in shock reflection: theoretical limit for regular reflection α_e, experimentally observed onset of Mach reflection α_0; and condition for sonic outflow α_s; *vs.* inverse pressure ratio across incident shock front.

In most respects, experimental observations support the regular reflection theory. When a plane shock wave is caused to impinge on a rigid wall at any angle smaller than α_e, a reflected shock does appear, intersecting the incident shock at the wall, of strength and angle of reflection in good agreement with the theoretical predictions (cf. Smith 1945).

Extension of such experiments to larger angles of incidence reveals that beyond some limiting value α_0, which is very close to (but always slightly larger than) the theoretical value α_e, the intersection point leaves the wall, and occurs instead at some finite distance from it. This intersection is then joined to the wall by a third shock, called the stem, and also by a slipstream, or contact surface, which trails behind the pattern, thus forming the well-known Mach configuration (see figure 3). The three-shock interaction occurring in this pattern has also been studied theoretically. This theory presumes that the pressure changes and flow deflections experienced by the gas passing through the pattern immediately above the 'triple point' (i.e. the intersection of the incident, reflected, and stem shocks) are the same as that for the gas passing immediately below it, although the exit velocities may in general be of different magnitude (as evidenced by the slipstream). Again, uniformity of the angular regions of flow is assumed, restricting the consideration to the immediate neighbourhood of the intersection.

2. The shock reflection paradox

Two areas of disagreement with the theories outlined above may be demonstrated experimentally. For convenient reference later, they are labelled I and II as follows:

I. The regular reflection process appears to persist for a small range of angles of incidence beyond the theoretical limit α_e. The intersection point remains at the wall for angles past α_e up to α_0, at which angle the Mach configuration begins. The interval $(\alpha_0 - \alpha_e)$ is measurable although small, being about two or three degrees for intermediate shock strengths (see figure 2).

II. Although the theoretical solutions for the Mach configuration (i.e. for the strength of the reflected shock at the triple point) are tolerable for strong shocks, they become increasingly inadequate for weaker incident shock strengths (see figure 4).

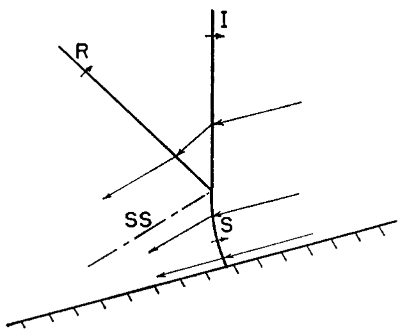

Figure 3. Mach reflection: intersection of reflected shock R with incident shock I is joined to wall by stem shock S. Slipstream (contact surface) SS separates two parallel outflows of different velocity, density and entropy.

Note that both of these problems involve essentially the same difficulty, namely the existence of a reflected shock that appears to violate the boundary requirements of the flow.

These discrepancies would not be anomalous if it could be shown that one of the assumptions of the theory is physically inadequate in the regions of discord, or, alternatively, that the experiments have insufficient resolution to reveal the true nature of the interaction in the small region to which the localized theory applies. Attempts to confirm such suspicions by shock reflection experiments have so far been inconclusive, however, as they are hampered by two inherent disadvantages. First, the angular interval of interest $(\alpha_0 - \alpha_e)$ is so small that, considering the resolution attainable in such experiments, it is difficult to make any systematic study of the angular dependence of a process within this range. The second, and probably more significant complication is introduced by the extraneous disturbances from the physical corner at which the reflection process begins. It is a curious feature that the outflow from the shock reflection pattern is subsonic for angles of incidence greater than a certain value α_s, which is just slightly smaller than α_e over the entire range of incident shock strength (see figure 2).

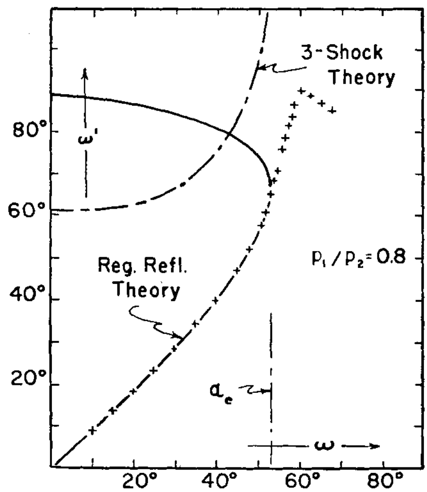

Figure 4. Shock reflection observations : experimental values agree well with regular reflection theory in the range $\alpha < \alpha_e$. At α_e, the observations leave the theoretical curve tangentially, and beyond α_0, in the Mach reflection region, disagree completely with the three-shock theory. (ω, ω' are the angles of incidence and reflection appropriate for pseudo-stationary description of the problem. For $\alpha < \alpha_0$, $\omega = \alpha$, $\omega' = \alpha'$.)

This means that the state of the region behind the reflected shock, which undoubtedly is the most interesting, is determined not only by the reflection process, but also by the nature of the boundaries at the 'corner' where the process first begins. Consequently, although some interesting density irregularities are observed in this region both for $\alpha_e < \alpha < \alpha_0$ and for the Mach patterns, it is difficult to determine whether these are essential parts of the reflection process or merely evidence of the corner interference. (In some respects it is tempting to consider that it is the presence of the corner which *causes* troubles I and II, but there is little experimental support for this point of view—cf. § 6.)

3. Shock wave refraction

Somewhat less ambiguous information on this subject has recently appeared in the course of an experimental study of another type of shock-wave interaction, namely the refraction of shock waves at interfaces between two gases (see Jahn 1956). This problem, while perhaps one degree more complex theoretically than the reflection problem, bears many similarities to it; and in some respects it is more tractable experimentally. The theory of shock refraction (Taub 1947; Polachek & Seeger 1951) assumes that a pseudo-stationary pattern will be formed, consisting of three shocks or two shocks and a Prandtl–Meyer wave (for our present purposes we need consider only the former case), and intersecting at a point on the gas interface (see figure 5). It is then required that the exit pressures, and the flow deflections via the upper and lower paths, be the same. Again, for given incident shock strength and gas combination, theoretical solutions

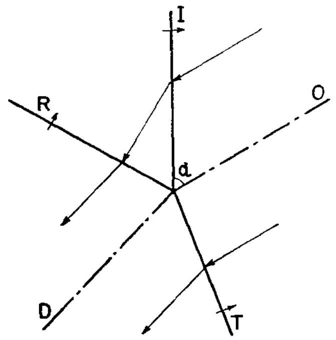

Figure 5. Regular refraction of a plane shock: shock I, incident on interface between two gases O, produces a reflected shock R and a transmitted shock T, the outflows from both of which are parallel to the deflected interface D.

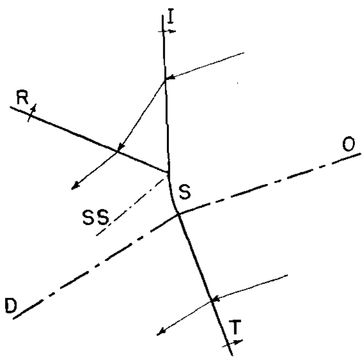

Figure 6. Mach refraction: intersection of reflected shock R and incident shock I is joined to interface by stem shock S. Again there is a slipstream SS from the intersection point.

can be obtained only for angles of incidence smaller than some critical value α_e. Furthermore, it appears experimentally that the assumed configuration persists, at least locally, beyond α_e up to some larger angle α_0 at which a 'Mach-refraction' pattern sets in (see figure 6).

Here too, then, there is a region of disagreement with the theory, akin to discrepancy I in the reflection problem. It is not unreasonable to suspect that the fundamental difficulty may be the same in both cases. Fortunately, the situation is more amenable to experimental study in the refraction case, since the interval $(\alpha_0 - \alpha_e)$ can be much larger for easily realizable conditions. Furthermore, in refraction experiments it is possible to change the arrangement of the physical boundaries to some extent, without necessarily changing the nature of the refraction conditions themselves; and thereby the features of the pattern which are consequences of the boundary conditions may be separated from those which are inherent in the shock interaction.

The results of refraction experiments pertinent to this problem are illustrated in figure 7 (plate 1). These are shock-tube photographs, taken through a 5-inch Mach–Zehnder interferometer set in parallel fringe adjustment, of the refraction of shock waves with pressure ratio 3·3 at an interface between air and methane. Figure 7(*a*) (plate 1) shows the configuration for $\alpha < \alpha_e$. Note that the three shocks are straight, the regions between them are uniform, and the intersection point is on the interface. Such a pattern, called a 'regular refraction', corresponds to the theoretical refraction solutions for this problem. Actually, the correspondence is encouragingly close; measurements of the shock strengths and angles of the configuration, taken from interferograms such as this, have been found to be in excellent agreement with the quantitative predictions of the theory.

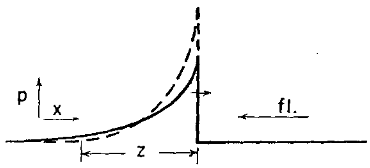

Figure 8. Pressure profiles of the reflected wave appearing in figure 7 (*b*) (plate 1). Shock front and following rarefaction zone Z are stronger near the interface (dotted profile) than further from it (solid profile).

However, as the angle of incidence is increased past α_e into the critical region $\alpha_e < \alpha < \alpha_0$, the pattern shown in figure 7(*b*) is observed. Although the shocks still intersect at a point on the interface, this is a more complex interaction in which the reflected wave is no longer straight, uniform in strength, or 'flat-topped'. Rather, it is considerably stronger near the interface, and less inclined to the flow there than it is further out into the field, and it is followed immediately by a very sharp rarefaction region, which also increases in severity near the interface. This composite reflected signal is thus a form of peaked shock wave (see figure 8).

The third interferogram in this series (figure 7(*c*), plate 1) shows the pattern for $\alpha > \alpha_0$. Here the reflection joins the incident shock wave a short distance away from the interface, and a stem shock and slipstream are now present in the pattern. Note that again the reflected wave is curved and non-uniform, and is followed by a region of rarefaction.

It is the non-uniform reflected shock and the sharp rarefaction zone seen behind it in figure 7(*b*) (plate 1) and, to a lesser extent, in figure 7(*c*) (plate 1) which particularly attract our attention in these configurations. A similar series of patterns, displaying zones of similar density variation can also be observed in shock-reflection experiments performed at the appropriate values of α (see figure 9, plate 2). These zones were noted by several earlier reflection investigators, but the unavoidable interference of the corner signals largely precluded evaluation of their significance to the interaction. The contribution of the refraction experiments lies in the possibility of eliminating the corner interference in the region of interest, so that the observed effects may be attributed directly to the shock interaction. It is this technique which permits us to attach significance to the sharp rarefaction zones seen in the refraction patterns displayed above, and then, because of the near equivalence of the two problems, to regard the corresponding zones of the reflection configurations in the same light. On this basis, all of the remarks in the following discussion pertain to both the reflection and refraction situations*.

4. The reflected wave

In addition to the more obvious features of the zone of interest around the reflected wave (i.e. the severity and direction of the density gradients and the variation of the strength of the reflected front itself along its length), two other pieces of information can be extracted from the interferograms. These are (1) that the region behind the reflected shock is one of subsonic flow, and (2) that the strength of the shock at the intersection point is very sensitively dependent on the angle of incidence.

The subsonic character of the flow behind the shock is revealed experimentally by the catching-up of the sonic signals from the corner (in those problems in which they are not exactly compensated to zero strength), or it can be deduced directly from measurements of the incident and reflected shock strengths and their inclinations to the flow (see Bleakney & Taub 1949). That this is a subsonic field suggests, of course, that any flow deflection processes taking place here will be continuous, relatively widespread adjustments, rather than the discontinuous, sharply localized processes found in supersonic flows. The significance of this point is emphasized by the work of Smith (1945), and Bargmann & Montgomery (1944), who have shown that a theory which assumes any supersonic variations in this region, such as other shocks, or Prandtl–Meyer fans, is in even poorer agreement with experiment than the theory postulating uniform flow.

It was mentioned that the strength of the reflected shock front increases rapidly along its length to some maximum value at the intersection point. This in itself suggests that some unusually severe process is taking place at the junction of the two shocks. It is also striking to plot the observed strength of the reflected shock at the interface against the angle of incidence, from which it appears that for angles immediately beyond α_e, the shock

* The attention of the reader is called to the recent work at Princeton of W. R. Smith (1956) in which reflection paradox I has also been examined in the light of a related shock interaction—in this case the oblique intersection of two shock waves of equal strength.

strength is extremely sensitive to α, and rises to a rather surprising value just before α_0 (see figures 4 and 10). For angles beyond α_0, the reflected shock strength falls off rapidly with increasing α, suggesting that the departure of the intersection point from the wall (or interface) relieves the situation which precipitated the sharp increase at α_e. That this is indeed the case may be seen in figure 7(*c*) (plate 1). In this well-developed Mach pattern, the slipstream from the intersection, which assumes the direction of the local flow velocity, is not parallel to the interface, indicating that the reflected shock has not had to deflect the flow through so great an angle as in the regular configurations.

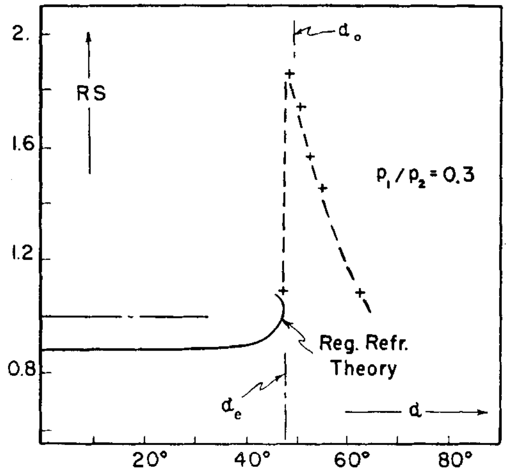

Figure 10. Shock refraction observations for $\alpha > \alpha_e$: strength of reflected shock front *RS* (pressure behind/pressure ahead) *vs.* angle of incidence α.

It appears then that we are dealing here with a transition process which prevails only in the brief angular interval between the degeneration of the regular pattern at α_e and the onset of the Mach configuration at α_0. This transition pattern is somewhat hybrid; it is 'regular' in the respect that the intersection point occurs on the boundary, but certain features of it, apparently associated with the non-uniform reflected shock and the subsonic rarefaction field following it, are somehow incompatible with the regular theory.

5. Interpretation of the observed reflected wave

Some insight into the significance of the anomalous reflected wave observed above may be gained from examination of other, more familiar gas-dynamic situations in which similar flow deflection requirements arise. For this purpose it is helpful to recognize that the limit α_e, at which these effects first appear, arises as a consequence of the well known 'maximum deflection' condition of oblique shocks (see, for example, Liepmann & Puckett 1952). At α_e, the reflected shock is of that optimum strength and corresponding inclination to the flow which accomplishes the maximum possible deflection of that flow. Beyond α_e, the gas outflow from the incident shock is such that no stationary oblique shock can re-deflect it parallel to the wall. The situation at the extreme angle in the refraction problem, while complicated by the adjacent conditions for the transmitted shock, is fundamentally the same. Here there are two shocks (reflected and transmitted) standing in two converging uniform streams. Under the requirement that the exit pressures be the same, there is just one arrangement of strengths and accompanying positions of these shocks by which the maximum deflection of the converging flows is accomplished. At α_e, this optimum configuration has been established, and beyond it some new process must occur.

An analogous flow deflection limit arises in the more familiar problem of the supersonic wedge aerofoil (figure 11, plate 3). Here, the shock tube is used as a transient wind tunnel. The incident shock has passed entirely over the model (from right to left), and we are observing the behaviour of the aerofoil in the 'steady' supersonic flow set up by that shock. (In this case, of course, a boundary layer develops along the aerofoil surface, but it is of zero thickness at the leading edge and does not fundamentally change the nature of the deflection problem.) Over a certain range of conditions, the deflection of the flow parallel to the aerofoil surfaces is accomplished abruptly by two straight bow shocks, attached to the nose of the body (see figure 11 (*a*), plate 3). (To emphasize the analogy this might be called a 'regular' deflection.) Beyond a certain limiting body angle (or below a certain flight velocity), however, it becomes impossible for any stationary oblique shock to deflect the inflow adequately. In this case, the bow shocks are observed to become stronger near the leading edge, and less inclined to the flow there. These stronger shocks produce subsonic, rather than supersonic, regions behind them containing expansions which are most severe behind the strongest portions of the shocks (figure 11 (*b*), plate 3). As the limit is exceeded further, the bow shocks fuse into one continuous front, detach from the leading edge, and advance into the oncoming flow (figure 11 (*c*), plate 3).

The interpretation of the processes seen in figures 11 (*b*) and 11 (*c*) (plate 3) seems straightforward: whatever part of the necessary flow deflection that is not accomplished abruptly through the bow shock is taken care of in a continuous fashion in the subsonic field that follows it. Such a subsonic field requires a stronger shock ahead of it than the supersonic field of figure 11 (*a*) (plate 3), and hence the shock must be more nearly normal to the inflow. If the deflection situation is made yet more severe, the region behind the shock becomes further subsonic, requiring a still stronger, more normal shock. Beyond the condition which produces a bow shock that is just normal to the flow at the wedge tip, there is no mechanism for further increasing the shock strength, other than for it to advance into the oncoming flow (thereby increasing its Mach number). The gap thus created between the wedge and the bow shock relieves the deflection requirement somewhat, and an equilibrium situation is established with the bow shock fixed a finite distance ahead of the wedge (figure 11 (c), plate 3). (This equilibrium position has been discussed by Laitone (1955).)

The similarities in the nature of the flow deflection requirements, and in the observed behaviour of the shocks and the regions behind them, in this problem and in the reflection–refraction cases are sufficiently striking to permit the inference that the mechanism in all three problems is fundamentally the same. In other words, the interpretation applied to the three aerofoil patterns in figure 11 (plate 3) should be equally appropriate for the regular, transition, and Mach patterns of the reflection–refraction problems. Such an analogy implies that the reflection and refraction patterns pass through the following sequence of phases as the angle of incidence is increased. From normal incidence up to $\alpha = \alpha_e$, the necessary re-deflection of the flow parallel to the boundary is accomplished abruptly by a reflected shock. Beyond α_e, no reflected shock is adequate to accomplish this re-deflection entirely, and the remainder of the process is brought about continuously in a subsonic rarefaction field that follows the reflected shock. Such a subsonic region requires a stronger reflected shock front, which in turn must be more normal to the outflow from the incident shock. As α is increased further beyond α_e, this outflow decreases in velocity, while simultaneously the deflection requirement becomes more severe, both of which force the reflected shock to be yet more normal to the flow. Consequently, at some larger angle α_0, the reflected shock has become just normal to the flow, and has no further recourse than to advance into it, much as the bow wave detaches from the aerofoil. In so advancing, the reflected shock overtakes a portion of the incident shock, thereby forming the characteristic Mach pattern. The departure of the intersection point from the interface relieves the deflection requirement somewhat, so that for a given angle of incidence an equilibrium condition is established with the intersection point fixed on the incident shock (in the pseudo-stationary frame of reference)*.

Superficially, this conception of the transition reflection process might not seem incompatible with the theory outlined earlier; but actually it does involve a feature which contradicts one of the basic assumptions. The flow deflection adjustment by the subsonic field following the shock has been described as a 'continuous' process, as it appears in the interferograms, to distinguish it from the abrupt change which occurs through the shock itself. The only portion of this field which is pertinent to the local theory, however, is that in the immediate neighbourhood of the intersection point, and here the term 'continuous' would not be appropriate. Rather, the width of this readjustment zone is observed to decrease, and the severity

* In a private communication, D. R. White has emphasized another requirement for the onset of Mach reflection, namely, that the stem shock must be strong enough to keep up with the incident shock: i.e. velocity of stem shock = (velocity of incident shock) $\times \sin \alpha_0$. This, in turn, implies a certain pressure behind the stem, and hence defines the strength of the reflected shock as a function of α_0 *if* the assumption of pressure continuity in the vicinity of the intersection point is valid.

of the density gradient in it to increase rapidly near the intersection point, apparently condensing into a discontinuous variation right at the vertex of the angle between the shock and the boundary. Indeed, on the basis of the proposed model, a discontinuity of this sort must be expected here, since, right at the boundary, the additional flow deflection must take place immediately. Such a variation would invalidate assumption (*b*) of the regular reflection theory; i.e. the changes in the flow in the angular region behind the reflection would not be negligible in comparison with those sustained through the shocks, regardless of how small a region about the intersection point is considered.

Sharply localized subsonic variations such as this can be found in other, more familiar aerodynamic situations. The same type of discontinuity occurs in the corresponding region of the supersonic aerofoil problem discussed above, for example, and also in the basic problems of subsonic flow over a sharp convex or concave corner (see figure 12). While supersonic flows

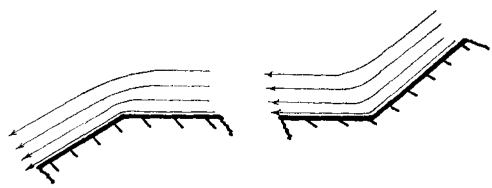

Figure 12. Subsonic flow over sharp corners. The abrupt deflections of the ideal-fluid streamlines at the corner are replaced, in real fluids, by regions of turbulence and separation.

can negotiate such abrupt changes in direction neatly, via oblique shocks and Prandtl–Meyer waves, theoretical calculations in the subsonic case become ambiguous right at the discontinuities in the boundaries, and infinite or zero velocities appear in the results. Physically, of course, such singularities in subsonic flows do not occur, but are replaced by regions of separation, eddy formation, or general turbulence, which reflect the non-ideal nature of the fluid involved, and which cannot be discussed on a non-viscous basis. However, these zones are normally quite small, and idealized theory can provide an adequate description of all the remaining flow field without any detailed consideration of the singular region.

Presumably, a similar dissipative process takes place in the close vicinity of the intersection point in the reflection problem, for here too there is evidence of a large velocity gradient. It would be highly interesting to observe directly the details of this process; but to date the experimental technique has not provided unambiguous description of the internal structure of this zone. The interferograms present information on the integrated density along the light path, and in this sense average out any three-dimensional irregularities or turbulence that might be taking place here. Only two-dimensional phenomena such as a cylindrical region of separation, or an eddy, could be recognized from the interferograms; and such effects, if they exist, apparently are confined to too small a neighbourhood to be distinguished from the shock intersections at the boundary. However, several uncorrelated cases of abnormal density variations in this region have been observed (see figure 13, plate 4). Such effects do not appear to be quantitatively reproducible, indicating, perhaps, that this is a somewhat unstable or turbulent region.

This inability to specify the microscopic nature of the flow-adjustment process does not qualify fundamentally the suggestion posed by interferograms like figures 7 (*b*) (plate 1) and 9 (*b*) (plate 2) regarding a theoretical resolution of the difficulty. The proposal is simply that an *ideal*-fluid theory of reflection can be reconciled with experiment in the transition zone $\alpha_e < \alpha < \alpha_0$ by the admission of a familiar type of subsonic singularity behind the reflected shock.

6. Subsonic singularities in Mach reflection

The second difficulty in the reflection problem—the failure of three-shock theory in application to weak-shock Mach configurations—can be approached from essentially the same point of view as the first; i.e. a search is made for experimental indication that one of the basic assumptions of the local theory is not realistic. Again, the refraction problem closely parallels the situation of interest; in fact, there is really no formal distinction between the three-shock Mach intersection, and the very special refraction problem of a shock incident obliquely on an interface separating air/air. In each case the assumed configuration and the required boundary conditions are the same (i.e. three shocks, intersecting at a point, out from which must come two parallel flows at the same pressure), and in each case, for $\alpha < \alpha_e$, two solutions appear, one of which is discarded as physically unreasonable. There is the distinction that in the air/air refraction case, one accepts the solution having a reflected wave of zero strength; for the Mach pattern, this branch is considered trivial. Nevertheless, the problems are sufficiently similar to suggest that if the general refraction pattern encounters flow-deflection difficulties beyond a certain angle of incidence, the Mach configuration may likewise suffer the same fate. The further inference then is that the mechanism of adjustment should also be similar, namely, a process like that outlined in § 5.

Closer examination of the three-shock Mach intersection reveals certain differences between the flow-deflection situation here, and that in the reflection or refraction problems. In the latter, we needed additional flow deflection away from the boundary. Here we find that the flows coming through the incident and Mach shocks are too divergent to be compensated by any moderately strong reflected shock, satisfying the outlet pressure conditions, and we look for a mechanism to complete the deflection toward the slipstream. (For weak incident shocks, the solutions of the three-shock theory place the reflected wave at an angle (ω') greater than 90° to the line connecting the corner and the triple point. In extreme cases, it is placed at an angle greater than 90° to the wall itself. To the present author, such configurations appear physically unreasonable, even in the local limit, for a pseudo-stationary process beginning at the corner.)

In these cases, the interferograms (see figures 7 (*c*), plate 1, and 9 (*c*), plate 2) show a subsonic compression following the reflection, and simultaneously a subsonic rarefaction following the Mach stem, both presumably helping in the flow and pressure readjustment process, and both presumably coalescing toward discontinuities right at the triple point. Since the three-shock theory applied locally to the Mach intersection does not admit such singularities, it would then follow that the observed reflected wave fronts would be in disagreement with the theory.

In committing ourselves to subsonic variations to reconcile the flow deflections in either the regular reflection or Mach interactions, we admittedly introduce certain other complications that need to be examined. These arise from the inherent non-uniformity of any finite transient subsonic field, and concern (*a*) the pseudo-stationary nature of the configurations and (*b*) the curvature of the streamlines of the flow. In the former matter, Bleakney & Taub (1949) have pointed out that for angles of incidence greater than α_s, another assumption of the theory—that the process is stationary in the coordinate system in which the intersection point is at rest—may no longer be justified, and the flow may be transient. If so, this would be sufficient grounds for discarding the theory in this range, and the observed discrepancies would no longer be a problem. However, on this point the accumulation of experimental data is most convincing that the transition patterns, and even the Mach patterns that occur at yet larger angles, are essentially pseudo-stationary interactions. The angles and shock strengths of the configurations appear to be independent of the absolute time, and the various field regions, including those that are subsonic, scale according x/t, y/t within the limits of observation. In view of this, and in view of the added experimental fact that the dissipative mechanism, whatever it may be, associated with the point of singularity in a real fluid is not sufficiently widespread to be observed at any reasonable time after inception, it seems more appropriate to maintain a pseudo-stationary approach when introducing the suggested alterations to the theoretical model.

The second complication involved in a subsonic region behind the reflected shock follows from the work of Taub (1953), who pointed out that if sonic signals from the corner can reach the intersection point, we must expect the reflection to be curved over its entire length. In general, the curvature of this shock will not be uniform, in which case the streamlines of the flow leaving it will also be curved, which clearly is at variance with the boundary condition for a straight wall. In this connection, the argument in favour of a subsonic singularity gains new strength. By analogies similar to those introduced above it could be shown that there is adequate precedent for imbuing this singularity with the property not only of deflecting the flow abruptly, but of straightening it as well. Again, the details of the 'uncurving' mechanism could presumably be inferred from the form of the density variation behind the shock, and the manner in which that variation condenses as it approaches the intersection point.

7. Conclusion

The present theories of shock reflection processes fail to account for (I), the persistence of a regular reflection process beyond a theoretical limiting angle of incidence α_e, and (II), the observed strength of the reflected shock in certain Mach reflection configurations. Direct reflection experiments are at a disadvantage in attempts to resolve these difficulties; but certain experiments on the shock-refraction process, the theoretical analysis of which is formally similar to the reflection problem, display, in corresponding regions of difficulty, pronounced subsonic variations behind the reflected shock. In the interval $\alpha_e < \alpha < \alpha_0$, it is observed that the shock is much stronger than might be expected near the gas interface, and is immediately followed by a steep, subsonic rarefaction there, whose function appears to be to complete the flow deflections, to apply the proper curvature to the streamlines, and, in some cases, to adjust the exit pressures. The severity and localization of this rarefaction zone increases sharply near the interface. In fact, the variation appears to coalesce toward a discontinuity at the intersection point, thereby suggesting a singularity in the theoretical problem.

The close theoretical correspondence of these refraction problems to the transition reflection ($\alpha_e < \alpha < \alpha_0$) and Mach-reflection situations, along with the observed similarities in the experimental observations, suggest resolution of the discrepancies in the latter by introduction of the same type of subsonic singularities.

It is expected that in any real fluid such singularities degenerate into some sort of dissipative process, such as eddies, separation, or general turbulence; but no positive identification of such effects on the photographs has been possible. Presumably, then, these processes are confined to very small regions; and hence their internal details should not be significant in the formulation of the problem. Rather, the description of the singularities could be based on observations, either theoretical or experimental, of the subsonic regions of adjustment at finite distances from the interaction, and the manner in which they condense as they approach the intersection point.

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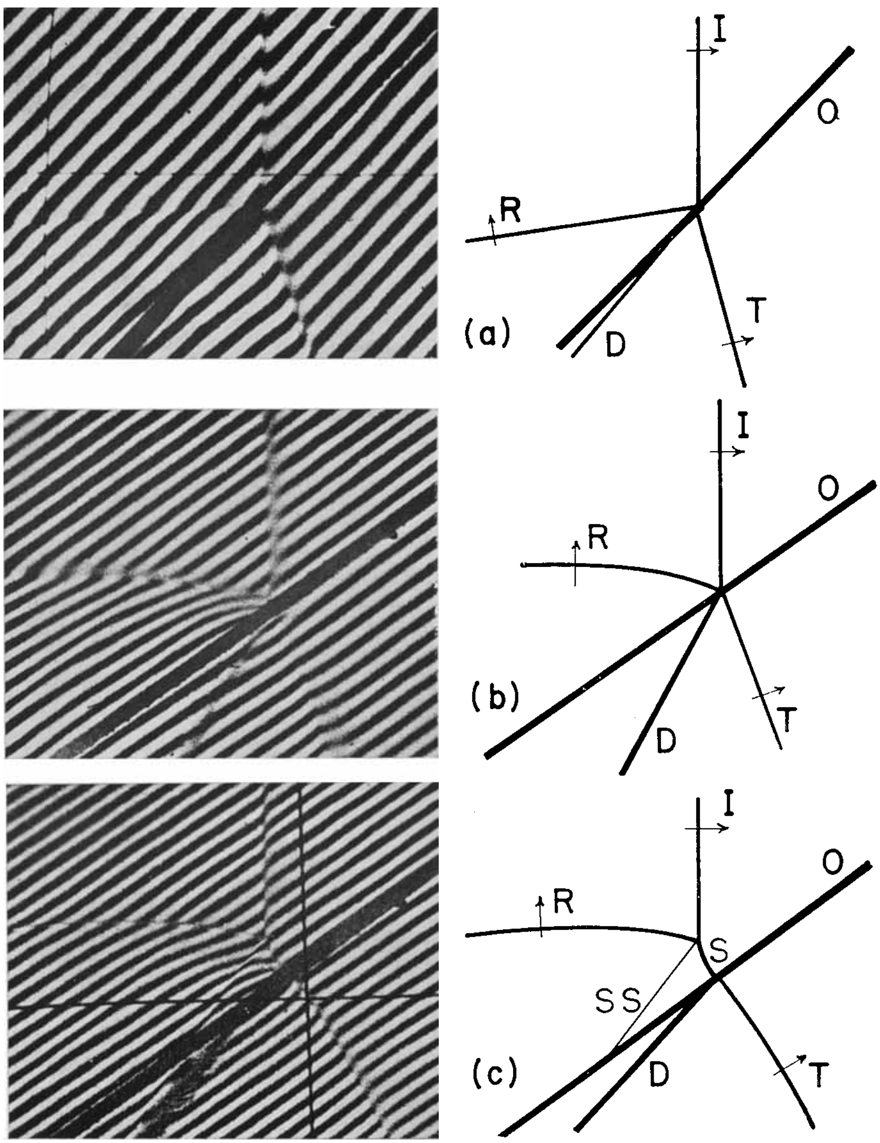

Figure 7. Shock refraction interferograms : configurations established by plane shock waves incident on interfaces between air and methane at angles (*a*) $\alpha < \alpha$; (*b*) $\alpha_e < \alpha < \alpha_0$; (*c*) $\alpha > \alpha_0$. (Symbols are same as in figures 5 and 6.)

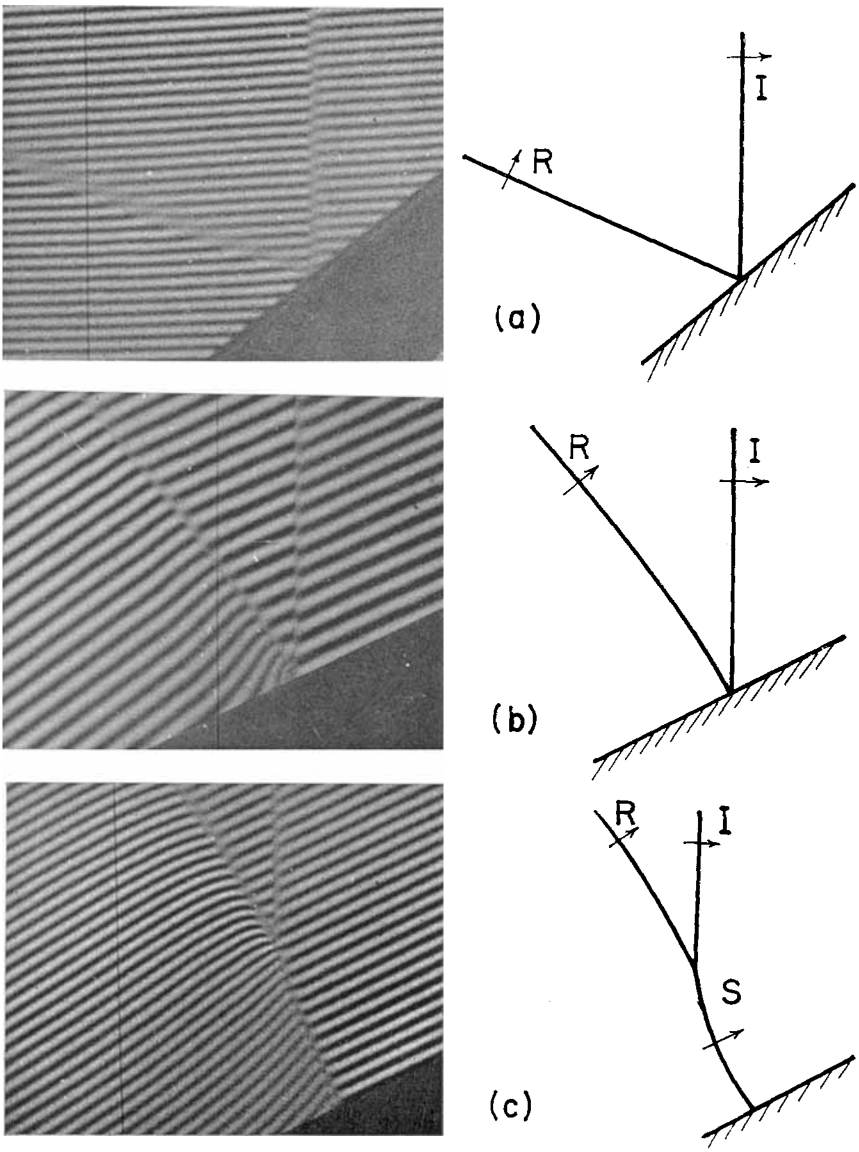

Figure 9. Shock reflection interferograms : configurations established by plane shock waves incident on a solid boundary at angles (*a*) $\alpha < \alpha_e$; (*b*) $\alpha_e < \alpha < \alpha_0$; (*c*) $\alpha > \alpha_0$. (Symbols are same as in figures 1 and 3.)

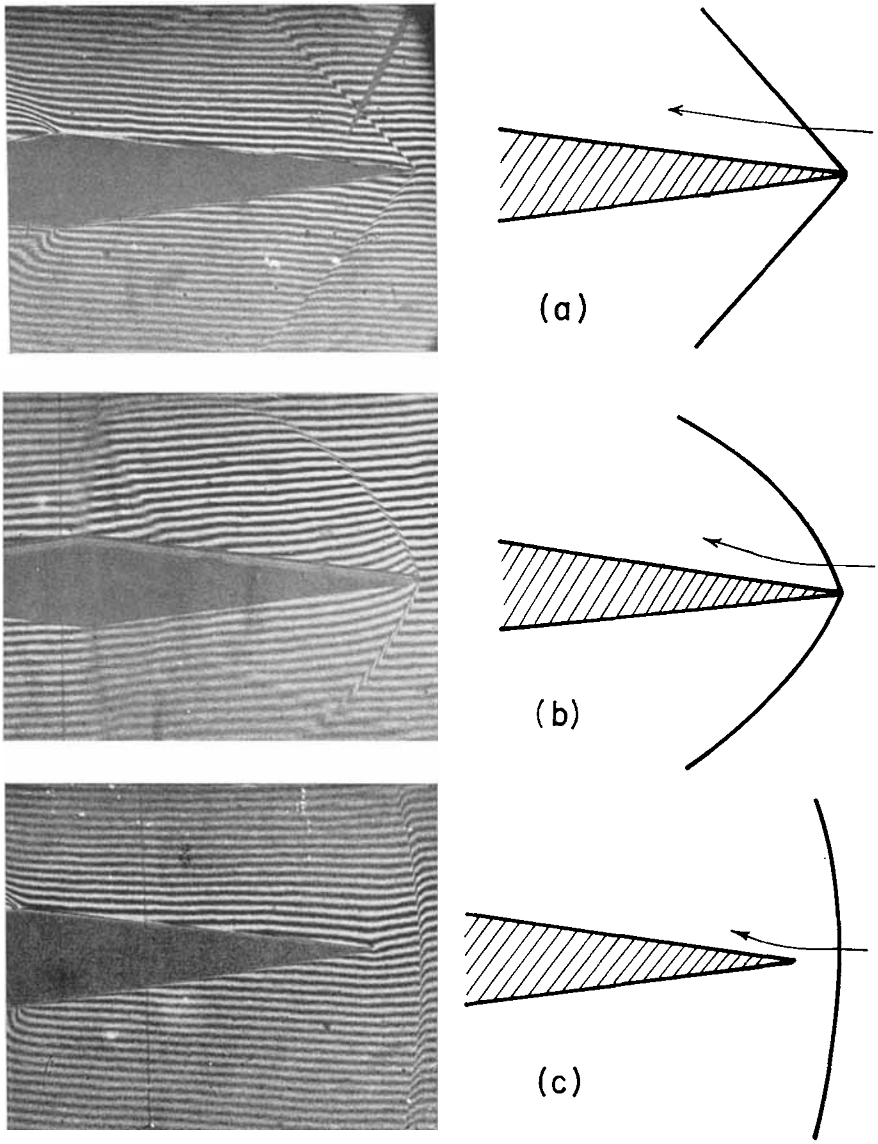

Figure 11. Diamond aerofoil at three-flight conditions : (*a*) straight attached bow shocks; (*b*) curved attached bow shocks; (*c*) detached bow shock.

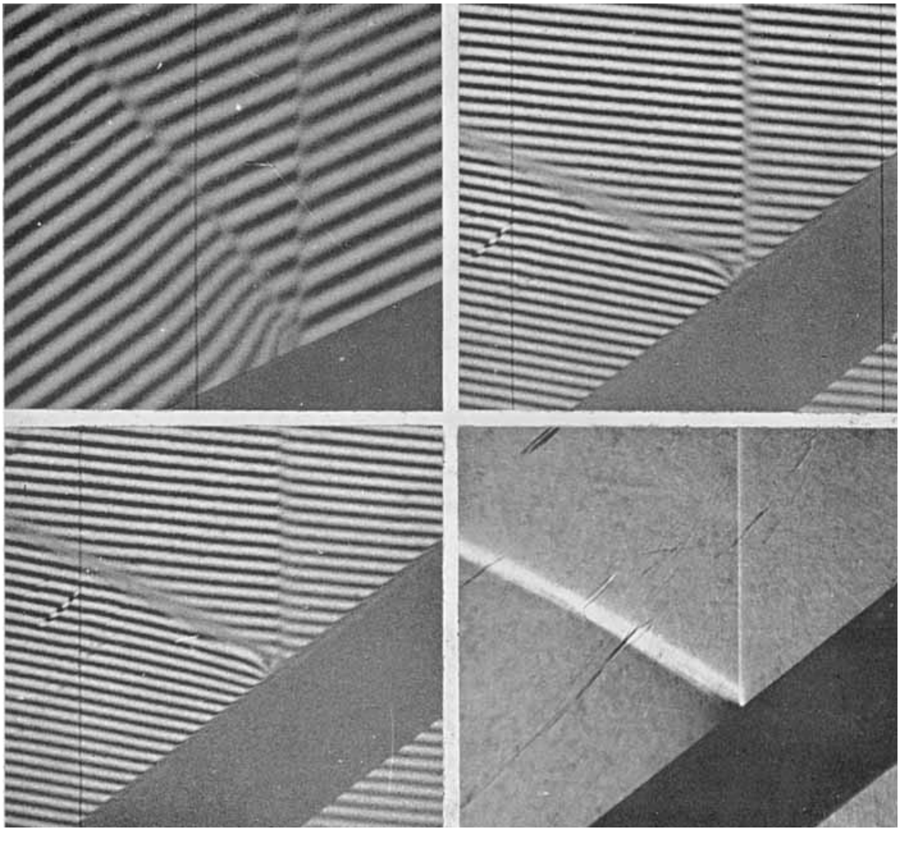

Figure 13. Isolated examples of severe density gradients behind the reflected shock. Three interferograms and one schlieren photograph of transition reflection processes.